Qual prep 2

Wednesday, September 22, 2021 3:29 PM

① Prove that a consistent finitely axiomatizable (possibly incomplete) theory T with less than continuum many completions must have a finitely axiomatizable completion.

② Say that a linear order is an **almost well-order** if every proper final segment of it is well-ordered. For example, ω^* is an almost well-order, but not a well-order. Prove that there are continuum many (non-isomorphic) countable almost well orders.

③ Consider the partial order $\mathbb{P} = (P(\omega), \subseteq)$
Show that:
1. If α is an ordinal that order-embeds into $\mathbb{P}$ then α is countable.
2. $\mathbb{R}$ order embeds into $\mathbb{P}$.

④. Call a total order **almost dense** if and only if it has no first or last element and there are no triples $x < y < z$ such that y is the only element between x and z. Prove that there are $2^{\aleph_0}$ non-isomorphic countable dense total orders.

⑤ Let $\mathcal{M}$ be a structure where $\phi(x,y)$ defines a linear order on an infinite set $X \subseteq \mathcal{M}$.
Given any linear order type τ, show that there are $\mathcal{N} \succeq \mathcal{M}$ and an infinite set $Y \subset \mathcal{N}$ of order type τ defined by $\phi(x,y)$.

⑥ Let X be any set and $f : P(X) \to P(X)$ be order preserving (i.e. for any $A, B \in P(X)$ if $A \subseteq B$ then $f(A) \subseteq f(B)$). Prove that there is a set $Y \in P(X)$ s.t $f(Y) = Y$.

⑦. Let A be a set totally ordered by $<$, and assume that in A there are no increasing or decreasing ω_1-sequences <u>and</u> no subsets isomorphic to the rationals. Prove that A is countable.