Putnam Club. Fall 2020

Hints for Problem session for October 28 (Polynomials).

1. discussed in class: apply Viete's theorem
2. discussed in class: $x = 0$ is a root of $P(x)$, write $P(x) = xP_1(x)$ and substitute it into the equation;
3. consider coefficients near x^9; use that the the polynomial of odd degree always has a root.
4. factorize the polynomial and use that the the polynomial of odd degree always has a root
5. try to substitute x such that $a_1x + b_1 = x$; what if you cannot find such x?
6. apply Viete's theorem; try to write some relations on r_1r_2 and r_3r_4;
7. use $a - b|P(a) - P(b)$;
8. use the formula for $P'(x)/P(x)$;
9. What you can say about the roots of this polynomial? How does the factorization of $P(x)$ look like?
10. show that operations commute and look on the few top coefficients;
11. substitute $x - 1$ and subtract the obtained equation from the original one;
12. the answer is any even n; what you can say about the set of real roots of $P(x)$?
13. use interpolation ideas;