1. Cartesian coordinates

- Let M be a point in the plane of triangle ABC. Prove that the centroids of the triangles MAB, MAC, and MCB form a triangle similar to triangle ABC.
- Find the locus of points P in the interior of a triangle ABC such that the distances from P to the lines AB, BC, and CA are the side lengths of some triangle.
- Let $A_1, A_2, \ldots, A_n$ be distinct points in the plane, and let m be the number of midpoints of all the segments they determine. What is the smallest value that m can have?
- Given an acute-angled triangle ABC with altitude AD, choose any point M on AD, and then draw BM and extend until it intersects AC in E, and draw CM and extend until it intersects AB in F. Prove that $\angle ADE = \angle ADF$.
- In a planar Cartesian system of coordinates consider a fixed point $P(a, b)$ and a variable line through P. Let A be the intersection of the line with the x-axis. Connect A with the midpoint B of the segment OP (O being the origin), and through C, which is the point of intersection of this line with the y-axis, take the parallel to OP. This parallel intersects PA at M. Find the locus of M as the line varies.
- Let $ABCD$ be a parallelogram with unequal sides. Let E be the foot of the perpendicular from B to AC. The perpendicular through E to BD intersects BC in F and AB in G. Show that $EF = EG$ if and only if $ABCD$ is a rectangle.
- Find all pairs of real numbers (p, q) such that the inequality

  $$\left|\sqrt{1-x^2} - px - q\right| \le \frac{\sqrt{2}-1}{2}$$

  holds for every $x \in [0, 1]$.
- On the hyperbola $xy = 1$ consider four points whose x-coordinates are x_1, x_2, x_3, and x_4. Show that if these points lie on a circle, then $x_1x_2x_3x_4 = 1$.

2. Complex coordinates

- Let $ABCDEF$ be a hexagon inscribed in a circle of radius r. Show that if $AB = CD = EF = r$, then the midpoints of BC, DE, and FA are the vertices of an equilateral triangle.
- Prove that in a triangle the orthocenter H, centroid G, and circumcenter O are collinear. Moreover, G lies between H and O, and $\frac{OG}{GH} = \frac{1}{2}$.
- On the sides of a convex quadrilateral $ABCD$ one draws outside the equilateral triangles ABM and CDP and inside the equilateral triangles BCN and ADQ. Describe the shape of the quadrilateral $MNPQ$.
- Let ABC be a triangle. The triangles PAB and QAC are constructed outside of the triangle ABC such that $AP = AB$, $AQ = AC$, and $\angle BAP = \angle CAQ = \alpha$. The segments BQ and CP meet at R. Let O be the circumcenter of the triangle BCR. Prove that AO and PQ are orthogonal.
- Let $A_1A_2\ldots A_n$ be a regular polygon with circumradius equal to 1. Find the maximum value of $\prod_{k=1}^{n} PA_k$ as P ranges over the circumcircle.

3. Circles and conics

- Consider a circle of diameter AB and center O, and the tangent t at B. A variable tangent to the circle with contact point M intersects t at P. Find the locus of the point Q where the line OM intersects the parallel through P to the line AB.

- On the axis of a parabola consider two fixed points at equal distance from the focus. Prove that the difference of the squares of the distances from these points to an arbitrary tangent to the parabola is constant.

- With the chord PQ of a hyperbola as diagonal, construct a parallelogram whose sides are parallel to the asymptotes. Prove that the other diagonal of the parallelogram passes through the center of the hyperbola.

- A straight line cuts the asymptotes of a hyperbola in points A and B and the hyperbola itself in P and Q. Prove that $AP = BQ$.

- Consider the parabola $y^2 = 4px$. Find the locus of the points such that the tangents to the parabola from those points make a constant angle ϕ.

- Let T_1, T_2, T_3 be points on a parabola, and t_1, t_2, t_3 the tangents to the parabola at these points. Compute the ratio of the area of triangle $T_1T_2T_3$ to the area of the triangle determined by the tangents.

- Three points A, B, C are considered on a parabola. The tangents to the parabola at these points form a triangle MNP (NP being tangent at A, PM at B, and MN at C). The parallel through B to the symmetry axis of the parabola intersects AC at L.
  (a) Show that $LMNP$ is a parallelogram.
  (b) Show that the circumcircle of triangle MNP passes through the focus F of the parabola.
  (c) Assuming that L is also on this circle, prove that N is on the directrix of the parabola.
  (d) Find the locus of the points L if AC varies in such a way that it passes through F and is perpendicular to BF.