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(1) Let $a_1, \ldots, a_n$ be n integers (not necessarily distinct). Prove that there exists a subset of these numbers whose sum is divisible by n.

(2) Let A and B be $n \times n$ matrices such that $AB = A + B$. Prove that $AB = BA$.

(3) Let x, y, and z be positive real numbers. Find the minimum value of $(x + 1/y)(y + 1/z)(z + 1/x)$.

(4) On a table there is a row of fifty coins, of various denominations (the denominations could be of any values). Alice picks a coin from one of the ends and puts it in her pocket, then Bob chooses a coin from one of the ends and puts it in his pocket, and the alternation continues until Bob pockets the last coin. Prove that Alice can play so that she guarantees at least as much money as Bob.

(5) Let A_n denote the number of ordered n-tuples of positive integers $(a_1, \ldots, a_n)$ such that $1/a_1 + \ldots + 1/a_n = 1$. Determine whether A_{10} is even or odd.

(6) Determine whether the following statement is true or false. For every finite set V of positive integers there exists a polynomial P with integer coefficients such that $P(1/n) = n$ for all n in V.

(7) Determine all positive integers relatively prime to all terms of the sequence $a_n = 2^n + 3^n + 6^n - 1$, $n \geq 1$.