ecurrences.

1. Find the number of subsets of $\{1, 2, \ldots, n\}$ that contain no two consecutive elements of $\{1, 2, \ldots, n\}$.

2. Determine the maximum number of regions in the plane that are determined by n "vee"s. A "vee" is two rays which meet at a point. The angle between them is any positive number.

3. Define a *domino* to be a 1×2 rectangle. In how many ways can an $n \times 2$ rectangle be tiled by dominoes?

4. (Putnam 1996) Define a *selfish* set to be a set which has its own cardinality (number of elements) as an element. Find, with proof, the number of subsets of $\{1, 2, \ldots, n\}$ which are *minimal* selfish sets, that is, selfish sets none of whose proper subsets are selfish.

5. Let $a_1, a_2, \ldots, a_n$ be an ordered sequence of n distinct objects. A *derangement* of this sequence is a permutation that leaves no object in its original place. For example, if the original sequence is $1, 2, 3, 4$, then $2, 4, 3, 1$ is not a derangement, but $2, 1, 4, 3$ is. Let D_n denote the number of derangements of an n-element sequence. Show that
$$D_n = (n-1)(D_{n-1} + D_{n-2}).$$

6. Let α, β be two (real or complex) numbers, and define the sequence $a_n = \alpha^n + \beta^n$ ($n = 1, 2, 3, \ldots$). Assume that a_1 and a_2 are integers with the same parity (both even or both odd). Prove that a_n is an integer for every $n \geq 1$.

7. Let t_1, t_2, t_3 be integers, and let $\lambda_1, \lambda_2, \lambda_3$ be real or complex numbers. Define the sequence $a_n = \lambda_1 t_1^n + \lambda_2 t_2^n + \lambda_3 t_3^n$ for $n = 0, 1, 2$. Prove that if a_0, a_1, and a_2 are integers then a_n is an integer for every $n \geq 0$.

8. Suppose that $x_0 = 18$, $x_{n+1} = \dfrac{10x_n}{3} - x_{n-1}$, and that the sequence $\{x_n\}$ converges to some real number. Find x_1.

9. (Putnam 2015-A2) Let $a_0 = 1, a_1 = 2$, and $a_n = 4a_{n-1} - a_{n-2}$ for $n \geq 2$. Find an odd prime factor of a_{2015}.