INTRODUCTORY PROBLEMS (09/23/15)

Warm-up

(From *Problem-Solving Strategies* by A. Engel)

1. If a and b are odd, then $a^2 + b^2$ is not a square.

2. All vertices of a convex pentagon are lattice points, and all of its sides have integral length. Show that its perimeter is even.

3. There are a white, b red, and c blue chips on the table. In one step, you choose two chips of different colors and replace them by a chip of the third color. In the end, only one chip remains. Prove that its color does not depend on the sequence of moves.

Some VT problems

(Chosen quasi-randomly from past exams on the official site.)

4. Find, and write out explicitly, a permutation $(p(1), p(2), \ldots, p(20))$ of $(1, 2, ..., 20)$ such that $k + p(k)$ is a power of 2 for $k = 1, 2, \ldots, 20$, and prove that only one such permutation exists. (To illustrate, a permutation of $(1, 2, 3, 4, 5)$ such that $k + p(k)$ is a power of 2 for $k = 1, 2, \ldots, 5$ is clearly $(1, 2, 5, 4, 3)$, because $1 + 1 = 2$, $2 + 2 = 4$, $3 + 5 = 8$, $4 + 4 = 8$, and $5 + 3 = 8$.)

5. An investor buys stock worth \$10,000 and holds it for n business days. Each day he has an equal chance of either gaining 20% or losing 10%. However in the case he gains every day (i.e. n gains of 20%), he is deemed to have lost all his money, because he must have been involved with insider trading. Find a (simple) formula, with proof, of the amount of money he will have on average at the end of the n days.

A few Putnam problems from last year

6. Suppose f is a function on the interval $[1, 3]$ such that $-1 \leq f(x) \leq 1$ for all x and $\int_1^3 f(x)dx = 0$. How large can $\int_1^3 \frac{f(x)}{x}dx$ be?

7. Let A be an $n \times n$ matrix whose entry in the i-th row and j-th column is $\frac{1}{\min(i,j)}$. Compute $\det(A)$.

UW Putnam Club

Meeting time: Wednesday 5–6:30pm, VV B 139.

Putnam competition: First Saturday in December (December 5, 2015). Two three-hour sessions of six problems each. Over 2,000 college students participate; there is also an official UW team (3 students).

Virginia Tech Regional Math Competition: 9–11:30 am, October 24, 2015, 7 problems. More than 600 contestants from over 100 schools. Kind of 'Putnam preparation', somewhat easier.

Common topics: Linear algebra, elementary number theory, calculus, combinatorics; emphasis on problem-solving.

Organizers:
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