PUTNAM CLUB MEETING: 09/22/14

Left-over from last time:

1. Solve the initial value problem

$$\frac{dy}{dx} = y\ln(y) + ye^x, \qquad y(0) = 1$$

Coloring (but no four-color theorem):

2. Warm-up: There are $2^4 \times 901^2 = 12988816$ ways to cover the 8×8 chessboard with dominoes (you don't have to prove this). How many ways are there to cover the chessboard with two diagonally opposite corners cut out?

3. Show that an 8×9 rectangle cannot be covered by 1×6 rectangles.

4. Show that an $a \times b$ rectangle can be covered by $1 \times n$ rectangle iff n divides a or n divides b.

5. A beetle sits on each square of a 9 x 9 chessboard. At a signal each beetle crawls diagonally onto a neighbouring square. Then it may happen that several beetles will sit on some squares and none on others. Find the minimal possible number of free squares.

Pigeonhole principle

6. Seven points are chosen in a regular hexagon of side length 1 unit. Show that some two of them are at most 1 unit apart.

7. 10 guests sit down at a round table at a dinner party. They notice that none of them is sitting at his or her assigned seat. Show that there is some rotation of their seating arrangement such that at least two people are in their assigned seats.

8. Given any five points on a sphere, show that some four of them must lie on a closed hemisphere. (Putnam 2002).

More pigeonhole problems to think about at home

9. (Putnam 1978) Let A be any set of 20 distinct integers chosen from the arithmetic progression $\{1, 4, 7, ..., 100\}$. Prove that there must be two distinct integers in A whose sum if 104.

10. Show that there exists a Fibonacci number that is divisible by 2014. (The Fibonacci sequence is defined by $F_0 = F_1 = 1$, $F_{n+1} = F_n + F_{n?1}$.)

11. Prove that there exist an integer n such that the first four digits of 2^n are 2, 0, 1, 4.

12. (IMO 1972.) Prove that from ten distinct two-digit numbers, one can always choose two disjoint nonempty subsets, so that their elements have the same sum.

13. Prove that every convex polyhedron has at least two faces with the same number of edges.