INTRODUCTORY PROBLEMS (09/20/16)

1. The decimal form of N consists of $243 = 3^5$ ones, and no other digits. Show that N is divisible by 243.

2. How many different necklaces can be made using seven beads? All beads are different color, two necklaces are considered the same if one can be obtained from the other by turning and possibly flipping it over.

3. Does there exist a tetrahedron such that each edge is a side of an obtuse angle on at least one of the two adjoint faces?

Math competitions

4. (UW UMC'16, #3) Find a simple general formula for
$$1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n!$$

5. (Based on VT'01, #4): Consider all positive integers k such that the greatest integer not exceeding $\sqrt{k}$ divides k. These integers form the following sequence (in order): $1, 2, 3, 4, 6, 8, 9, 12, \ldots$. What is the 1000th number in this sequence?

6. (Based on VT'99, #1): $f(x)$ is a continuous function such that $f(x) = f(x+1)$ for all x (that is, f is periodic). You are told that $\int_0^1 f(x)dx = 1999$. Find
$$\int_0^1 \int_0^x f(x+y)\, dy\, dx.$$

7. (Putnam'15, #B1): Let f be a three times differentiable function such that f has at least five distinct real zeros. Prove that
$$f + 6f' + 12f'' + 8f'''$$
has at least two distinct real zeros.

UW Putnam Club

Meeting time: Tuesday 5–6:30pm, VV B337.

Putnam competition: First Saturday in December (December 3, 2016). Two three-hour sessions of six problems each. Over 2,000 college students participate; there is also an official UW team (3 students).

Virginia Tech Regional Math Competition: 9–11:30 am, October 22, 2016, 7 problems. More than 700 contestants from around 100 schools. Kind of 'Putnam preparation', somewhat easier.

UW Undergraduate Math Competition: Spring 2017 (probably April).

Typical topics: Linear algebra, elementary number theory, calculus, combinatorics; emphasis on problem-solving.