# Some Probability Problems

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1. (Putnam and beyond, page 311) Consider n indistinguishable balls randomly distributed in m boxes. What is the probability that exactly k boxes remain empty?

2. For a polynomial $f(x) = x^2 + ax + b$ where $a, b \in [0, 1]$ with uniform distribution, what is the probability that $f(x)$ has two real roots?

3. Randomly chose two intervals within $[0, 1]$. (How to make this precise?) What is the probability that they overlap?

4. (Putnam and beyond, Problem 912) What is the probability that a permutation of the first n positive integers has the numbers 1 and 2 within the same cycle.

5. (Putnam 2002 B1) Shanille O'Keal shoots free throws on a basketball court. She hits the first and misses the second, and thereafter the probability that she hits the next shot is equal to the proportion of shots she has hit so far. What is the probability she hits exactly 50 of her first 100 shots?

6. (Putnam 1993 B3) Two real numbers $x$ and $y$ are chosen at random in the interval (0, 1) with respect to the uniform distribution. What is the probability that the closest integer to $x/y$ is even? Express the answer in the form $r + s\pi$, where $r$ and $s$ are rational numbers.