1. Let $a, b > 1$ be integers and $g := \gcd(a, b)$ its greatest common divisor. Show that if $a = g \cdot q_a$ and $b = g \cdot q_b$ then q_a and q_b are relatively prime.

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2. Show that for any pair of non negative integers a and b

$$a \cdot b = \gcd(a, b) \cdot \operatorname{lcm}(a, b).$$

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3. Find $\gcd(1000, 625)$

   (a) using the Euclidean Algorithm
   and

   (b) by factorization.

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4. (a) If p is prime, show that the largest power of p dividing n! is

$$\sum_{j=1}^{\log_p n} \left\lfloor \frac{n}{p^j} \right\rfloor = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \dots$$

   or

$$\frac{n - \sigma_p(n)}{p - 1}$$

   where $\sigma_p(n)$ denotes the sum of the base p digits of n.

   (b) 1000! has a lot of final zero digits. Use (a) to find how many are there.

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5. (a) Given two non negative relatively prime integers a an b, show that if x_0, y_0 is a particular solution of the Diophantine equation $ax + by = m$ then, any other solution is of the form

$$\begin{cases} x = x_0 + b\kappa \\ y = y_0 - a\kappa \end{cases}$$

   for some integer κ.

   (b) Use (a) to describe the solution set for the general linear Diophantine equation $ax + by = m$ when a and b are arbitrary non negative integers.

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6. Solve

$$\left.\begin{aligned} x &\equiv 1 \mod 3 \\ x &\equiv 2 \mod 5 \end{aligned}\right\}$$